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Analysis of infinite *d*-dimensional networks – Capacitance between two adjacent nodes

Jihad H. Asad*

Dep. of Physics, Tabuk University, P.O. Box 741, Dhiba Road, Tabuk 71491, Saudi Arabia

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A B S T R A C T

This work showed that infinite *d*-dimensional networks consisting of identical capacitors each of capacitance C can be analyzed using basic concepts of physics. In this work we have showed that the equivalent capacitance C_{eq} between any two adjacent nodes in the infinite *d*-dimensional networks consisting of identical capacitors, is equal to dC where d is the dimension of the infinite network (i.e., $d = 1,2,3,\ldots$). The results obtained here are in an excellent agreement with previous studied carried out.

1. Introduction

Analysis of infinite electrical networks consisting of resistors is a basic goal for physicists and electrical engineering. Kirchhoff's [1] was one the first physicists to formulate the study of electric networks. During the past sixty years many approaches have been developed and used in studying this problem. These approaches can be classified mainly into three main methods: The first method is based on superposition of current distribution [2,3] where in this method numerical results for the resistances between the origin and any other lattice site in an infinite square, triangular, hexagonal, and simple cubic networks consisting of identical resistors have been presented. The second method is based on random walk, where Monwhea Jeng [4] introduced a mapping between random walk problems and resistor network problems, where his method was used to calculate the effective resistance between any two sites in an infinite two-dimensional square lattice of unit resistors.

Finally, twelve years ago, an important and educational method was presented by Cserti [5,6] where this method is based on so-called Lattice Green's Function (LGF). The importance of this method comes from the fact that it is valid for all perfect lattices, in addition applying a perturbed (i.e., a resistor is removed from the infinite perfect network) study to square and Simple Cubic (SC) lattice for the fist time. Many studies have been carried out using Cserti's method [7–11]. In these studies further extensions have been done including removing two resistors from the perfect square and SC networks in addition to an experimental part that has been done for the first time.

Little attention has been paid for analyzing infinite networks consisting of capacitors. Asad et al. [12–16] applied Cserti's method and superposition of charge distribution method to infinite networks consisting of identical capacitors where a study for the infinite square and SC networks consisting of identical capacitors has been studied for both the perfect and perturbed case.

2. Infinite *d*-dimensional networks

Our goal in this section is to find the equivalent capacitance C_{eq} between any two adjacent nodes in an infinite *d*-dimensional networks consisting of identical capacitors each of capacitance C, where $d = 1,2,3,\ldots$ and the network is periodic, perfect and infinite in all *d*-dimensions. In other words we aim to generalize our previous study [17] which has been carried for analyzing infinite 2D networks of identical capacitors.

To illustrate the problem simply, consider an infinite 2D square network consisting of equal capacitors as shown in Fig. 1. It is clear that number of capacitors connected to each node is equal to 4. For a *d*-dimensional network, this number will be equal to $2d$. To find the equivalent capacitance C_{eq} between any two adjacent nodes in the *d*-dimensional network we will use the superposition of charge distribution in addition to symmetry of the network.

* Tel.: +966 553272187.
E-mail addresses: jhasad1@yahoo.com, jasad@ut.edu.sa.

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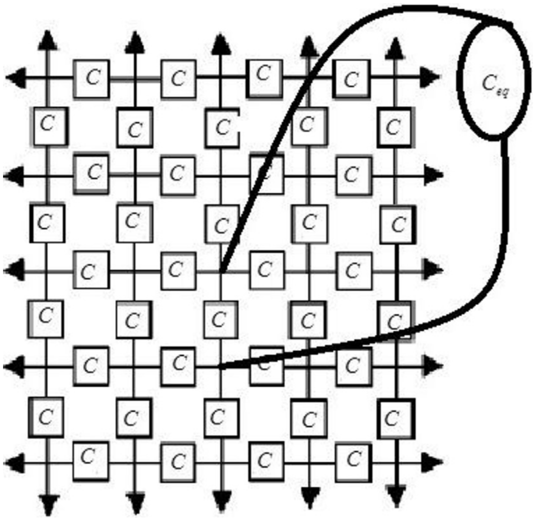

Fig. 1. An infinite 2D square network consisting of equal capacitors each of capacitance C.

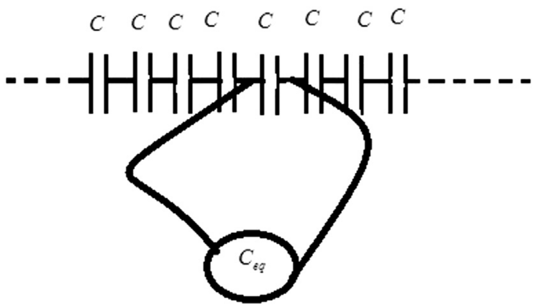

Fig. 2. An infinite chain of identical capacitors ($d = 1$).

Now, consider a test charge Q injected at node 1 and then extract it from node 2 which is adjacent to node 1. The two nodes are at a zero-potential boundary at infinity. According to Kirchhoff's laws and due to the symmetry of the d-dimensional network, the charge entering each of the $2d$ capacitors connected to node 1 is equal to $Q/2d$ of the injected charge Q. Whereas each of the $2d$ capacitors connected to node 2 will receive $-Q/2d$ of the extracted charge Q.

Applying the superposition principle, the total charge flowing in the capacitor C connected between node 1 and node 2 will be equal to $(2Q/2d) = Q/d$. This will leads to a voltage drop, across the capacitor C connected between node 1 and node 2, be equal to $V_{\text{drop}} = (Q/d)/C$. Thus, the equivalent capacitance between node 1 and node 2 can be obtained by:

$$C_{\text{eq}} = \frac{Q}{V_{\text{drop}}};$$

$$C_{\text{eq}} = dC. \tag{1}$$

The last equation is our remarkable, simple and elegant result. It can be applied to any infinite d-dimensional network consisting of identical capacitors. To check Eq. (1) we consider the trivial case where $d = 1$, this means that we have an infinite chain of capacitors each with capacitance C as shown in Fig. 2, the result is $C_{\text{eq}} = C$.

It is worth mentioning that for both $d = 2{,}3$ respectively then the equivalent capacitance is $C_{\text{eq}} = 2C$, $3C$ respectively. The obtained results are in an exact agreement with those obtained using other methods [12–17].

3. Conclusion

In this work, we have found the equivalent capacitance C_{eq} between any two adjacent nodes in an infinite d-dimensional networks consisting of identical capacitors. We have used in this work, basic concepts of physics (i.e., Kirchhoff's laws, symmetry and superposition principle). The obtained results are in an exact agreement with those obtained previously using more complex methods (i.e., LGF method in addition to superposition of charge distribution method). Finally, the content of this work is helpful for physicists and electric circuit design.

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